

Investigating Geometry

Mathematics Curriculum Framework

Revised 2004
Amended 2006

Course Title: Investigating Geometry
 Course/Unit Credit: 1
 Course Number:
 Teacher Licensure: Secondary Mathematics
 Grades: 9-12

Investigating Geometry

Students in this course will be accountable for the SLEs in Geometry. However, the instructional strategies to be utilized focus on individual and/or cooperative group work, the discovery method, hands-on activities, and problem solving strategies involving manipulatives and technology.

Strand	Standard
Language of Geometry	
	1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.
Triangles	
	2. Students will identify and describe types of triangles and their special segments. They will use logic to apply the properties of congruence, similarity, and inequalities. The students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real world situations.
Measurement	
	3. Students will measure and compare, while using appropriate formulas, tools, and technology to solve problems dealing with length, perimeter, area and volume.
Relationships between two- and three-dimensions	
	4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
Coordinate Geometry and Transformations	
	5. Students will specify locations, apply transformations and describe relationships using coordinate geometry.

* denotes amended changes to the framework

Language of Geometry

Content standard 1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.

LG.1.G.1	Define, compare and contrast <i>inductive reasoning</i> and <i>deductive reasoning</i> for making predictions based on real world situations <ul style="list-style-type: none"> • <i>venn diagrams</i> • <i>matrix logic</i> • <i>conditional statements</i> (statement, <i>inverse</i>, <i>converse</i>, and <i>contrapositive</i>) • <i>*figural patterns</i>
LG.1.G.2	Represent <i>points</i> , <i>lines</i> , and <i>planes</i> pictorially with proper identification, as well as basic concepts derived from these undefined terms, such as segments, rays, and <i>angles</i>
LG.1.G.3	Describe relationships derived from geometric figures or figural patterns
LG.1.G.4	Apply, with and without appropriate technology, definitions, <i>theorems</i> , properties, and <i>postulates</i> , related to such topics as <i>complementary</i> , <i>supplementary</i> , <i>vertical angles</i> , <i>linear pairs</i> , and angles formed by <i>perpendicular lines</i>
LG.1.G.5	Explore, with and without appropriate technology, the relationship between angles formed by two lines cut by a <i>transversal</i> to justify when lines are <i>parallel</i>
LG.1.G.6	Justify conclusions reached by deductive reasoning *State and prove key basic theorems in geometry (i.e., the Pythagorean theorem, the sum of the measures of the angles of a triangle is 180° , and the line joining the midpoints of two sides of a triangle is parallel to the third side and half it's length

Suggested activities may include but are not limited to the following:

* Modeling Venn diagrams to show class makeup (string, hoops, chart paper and colored dots)
* Establishing scenarios that can be examples of intuition, inductive or deductive thinking
* Using physical models to represent points, lines, planes, segments and rays
* Using patty paper to find midpoints, bisectors, perpendicular and angle bisectors
* Using compass and straight-edge to perform constructions
* Using geoboards to investigate <ul style="list-style-type: none"> • angles • parallel lines • perpendicular lines • polygons

Triangles

Content Standard 2. Students will identify and describe types of triangles and their special segments. They will use logic to apply the properties of congruence, similarity, and inequalities. The students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real world situations.

T.2.G.1	Apply <i>congruence</i> (SSS ...) and <i>similarity</i> (AA ...) correspondences and properties of figures to find missing parts of geometric figures and provide logical justification
T.2.G.2	Investigate the measures of segments to determine the existence of triangles (<i>triangle inequality theorem</i>)
T.2.G.3	Identify and use the special segments of triangles (<i>altitude, median, angle bisector, perpendicular bisector, and midsegment</i>) to solve problems
T.2.G.4	Apply the Pythagorean Theorem and its converse in solving practical problems
T.2.G.5	Use the <i>special right triangle</i> relationships (30° - 60° - 90° and 45° - 45° - 90°) to solve problems
T.2.G.6	Use <i>trigonometric ratios</i> (<i>sine, cosine, tangent</i>) to determine lengths of sides and measures of angles in right triangles including <i>angles of elevation</i> and <i>angles of depression</i>
T.2.G.7	*Use similarity of right triangles to express the sine, cosine, and tangent of an angle in a right triangle as a ratio of given lengths of sides

Suggested activities may include but are not limited to the following:

* Using patty paper or a compass and straight-edge to investigate special segments of triangles
* Using segment manipulatives to determine the existence of triangles (spaghetti, straws, toothpicks, ...)
* Using an object and its projection to investigate the dimensions of the object and its shadow
* Using geoboards to investigate the Pythagorean Theorem
* Using a compass and straight-edge, patty paper and/or plastic transparent reflectors to discover the relationship of the sides of special right triangles

Measurement

Content Standard 3. Students will measure and compare while using appropriate formulas, tools, and technology to solve problems dealing with length, perimeter, area and volume.

M.3.G.1	Calculate probabilities arising in geometric contexts (Ex. Find the probability of hitting a particular ring on a dartboard.)
M.3.G.2	Apply, using appropriate units, appropriate formulas (<i>area, perimeter, surface area, volume</i>) to solve application problems involving <i>polygons, prisms, pyramids, cones, cylinders, spheres</i> as well as composite figures, expressing solutions in both exact and approximate forms
M.3.G.3	Relate changes in the measurement of one <i>attribute</i> of an object to changes in other attributes (Ex. How does changing the <i>radius</i> or height of a cylinder affect its surface area or volume?)
M.3.G.4	Use (given similar geometric objects) proportional reasoning to solve practical problems (including scale drawings)
M.3.G.5	*Identify and apply properties of and theorems about parallel and perpendicular lines to prove other theorems and perform basic Euclidean constructions

Suggested activities may include but are not limited to the following:

* Determining the amount of paint needed to paint the classroom
* Determining the amount of border needed to trim the classroom
* Determining the number of soda cans needed to fill a box
* Using solid shapes to show the relationships of the volume of cones and cylinders, pyramids and rectangular prisms
* Determining the amount of air an official basketball holds
* Using physical models to show how the change in one measurement affects the area, perimeter, volume, or surface area
* Drawing blueprints for an actual building
* Constructing scale models

Relationships between two- and three-dimensions

Content Standard 4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

R.4.G.1	Explore and verify the properties of <i>quadrilaterals</i>
R.4.G.2	Solve problems using properties of polygons: <ul style="list-style-type: none"> • sum of the measures of the <i>interior angles of a polygon</i> • interior and <i>exterior angle measure of a regular polygon or irregular polygon</i> • number of sides or angles of a polygon
R.4.G.3	Identify and explain why figures <i>tessellate</i>
R.4.G.4	Identify the attributes of the five <i>Platonic Solids</i>
R.4.G.5	Investigate and use the properties of angles (<i>central and inscribed</i>) <i>arcs, chords, tangents, and secants</i> to solve problems involving <i>circles</i>
R.4.G.6	Solve problems using inscribed and <i>circumscribed</i> figures
R.4.G.7	Use <i>orthographic drawings</i> (top, front, side) and <i>isometric drawings</i> (corner) to represent three-dimensional objects
R.4.G.8	Draw, examine, and classify <i>cross-sections</i> of three-dimensional objects
R.4.G.9	*Explore non-Euclidean geometries, such as spherical geometry and identify its unique properties which result from a change in the parallel postulate

Suggested activities may include but are not limited to the following:

* Making conjectures about properties of quadrilaterals by constructing or drawing quadrilaterals, measuring angles, sides, and diagonals
* Using models of polygons to measure angles, make tables, and discover the formula for the sum of the measures of the interior angles of a polygon
* Using pattern blocks to discover which polygons will tessellate the plane
* Modeling Platonic solids and investigating attributes
* Using patty paper and geometric constructions to discover properties of angles, arcs, chords, tangents and secants of circles
* Building and drawing (top, side, front, and corner) sketches of buildings using isometric dot paper, cuisenaire rods and/or colored cubes
* Building different 3-D figures (prisms, pyramids, spheres, cylinders, and cones) with clay or using solids as molds for gelatin jigglers and slicing them with dental floss or string to examine the cross sections

Coordinate Geometry and Transformations

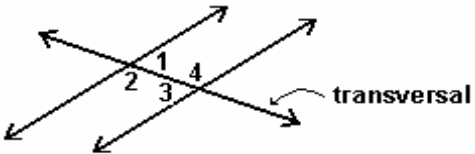
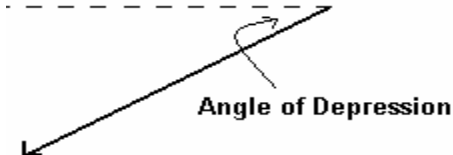
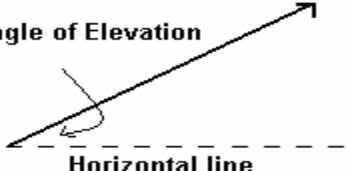
Content Standard 5. Students will specify locations, apply transformations and describe relationships using coordinate geometry.

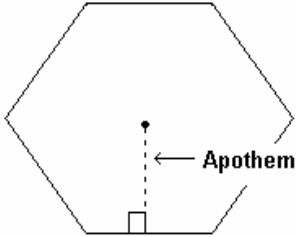
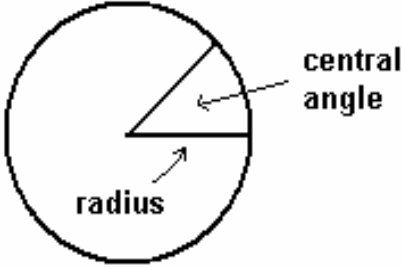
CGT.5.G.1	Use <i>coordinate geometry</i> to find the distance between two points, the <i>midpoint of a segment</i> , and the <i>slopes</i> of parallel, perpendicular, horizontal, and vertical lines
CGT.5.G.2	*Write the equation of a line parallel to a line through a given point not on the line
CGT.5.G.3	*Write the equation of a line perpendicular to a line through a given point
CGT.5.G.4	*Write the equation of the perpendicular bisector of a line segment
CGT.5.G.5	Determine, given a set of points, the type of figure based on its properties (<i>parallelogram, isosceles triangle, trapezoid</i>)
CGT.5.G.6	Write, in standard form, the equation of a circle given a graph on a coordinate plane or the center and radius of a circle
CGT.5.G.7	Draw and interpret the results of transformations and successive <i>transformations</i> on figures in the coordinate plane <ul style="list-style-type: none"> • <i>translations</i> • <i>reflections</i> • <i>rotations</i> (90°, 180°, clockwise and counterclockwise about the origin) • <i>dilations</i> (scale factor)

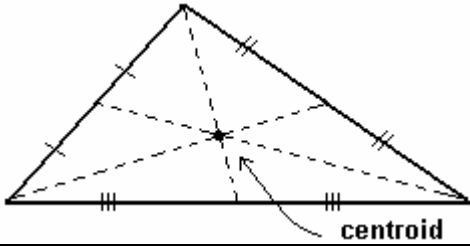
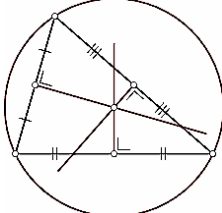
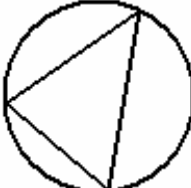
Suggested activities may include but are not limited to the following:

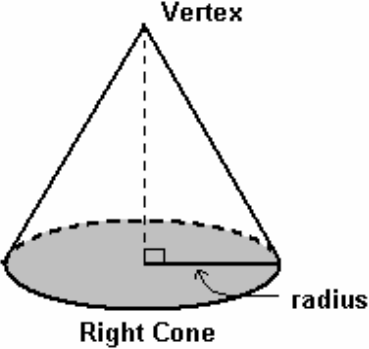
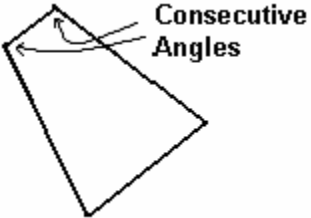
* Using geoboards to discover the relationships among polygons
* Using dynamic computer software and graphing calculator applications and plastic transparent reflectors to investigate relationships between lines and interpret results of transformations
* Using a plastic transparent reflector or compass and straightedge to find the perpendicular line and discover the relationship given a graph of a line on a coordinate plane

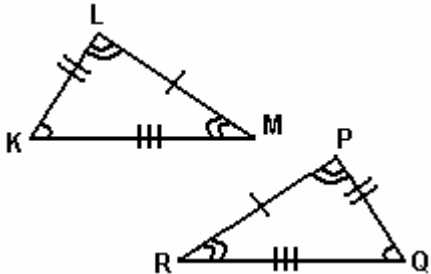
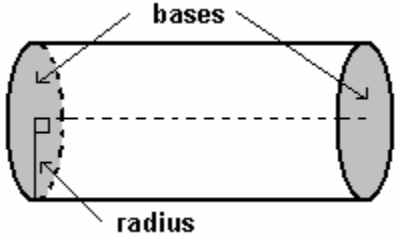
GEOMETRY Glossary

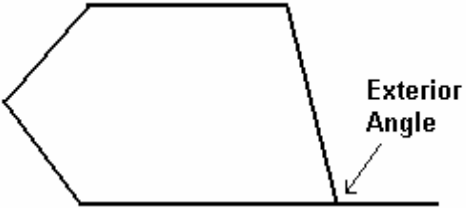
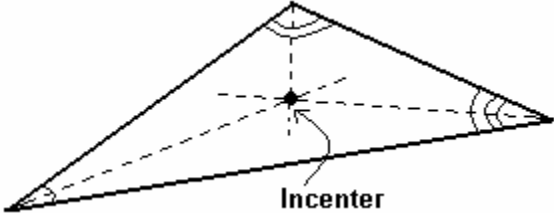
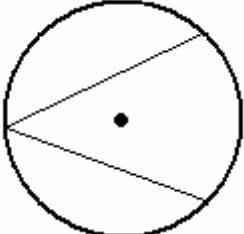
<i>Adjacent angles</i>	Two coplanar angles that share a vertex and a side but do not overlap
<i>Alternate interior angles</i>	<p>Two angles that lie on opposite sides of a transversal between two lines that the transversal intersects</p>  <p>The diagram shows two parallel lines intersected by a transversal. The transversal is labeled 'transversal' with an arrow. Four angles are labeled: 1 (top-right), 2 (top-left), 3 (bottom-left), and 4 (bottom-right). Angles 1 and 3 are alternate interior angles, as are angles 2 and 4.</p>
<i>Altitude of a triangle</i>	A perpendicular segment from a vertex of a triangle to the line that contains the opposite side
<i>Angle</i>	Two non-collinear rays having the same vertex
<i>Angle of depression</i>	<p>When a point is viewed from a higher point, the angle that the person's line of sight makes with the horizontal</p>  <p>The diagram shows a horizontal dashed line labeled 'Horizontal line'. A solid line representing the line of sight extends downwards and to the left from the right end of the horizontal line. The angle between the horizontal line and the line of sight is marked with a curved arrow and labeled 'Angle of Depression'.</p>
<i>Angle of elevation</i>	<p>When a point is viewed from a lower point, the angle that the person's line of sight makes with the horizontal</p>  <p>The diagram shows a horizontal dashed line labeled 'Horizontal line'. A solid line representing the line of sight extends upwards and to the right from the left end of the horizontal line. The angle between the horizontal line and the line of sight is marked with a curved arrow and labeled 'Angle of Elevation'.</p>

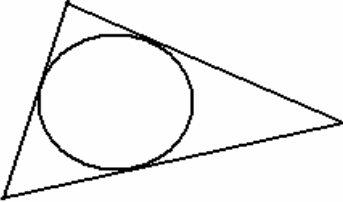
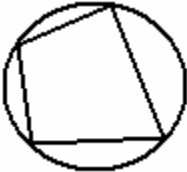
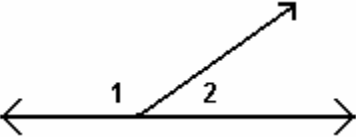
<i>Apothem</i>	<p>The distance from the center of a regular polygon to a side</p> 
<i>Arcs</i>	An unbroken part of a circle
<i>Area</i>	The amount of space in square units needed to cover a surface
<i>Attributes</i>	A quality, property, or characteristic that describes an item or a person (Ex. color, size, etc.)
<i>Biconditional</i>	A statement that contains the words "if and only if" (This single statement is equivalent to writing both "if p, then q" and its converse "if q then p.")
<i>Bisector</i>	A segment, ray or line that divides into two congruent parts
<i>Center of a circle</i>	The point equal distance from all points on the circle
<i>Central angle</i>	<p>An angle whose vertex is the center of a circle (Its measure is equal to the measure of its intercepted arc.)</p> 

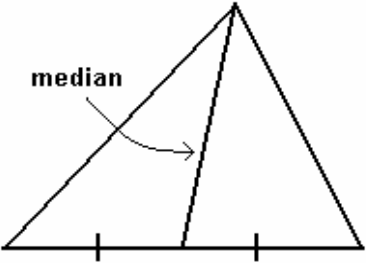
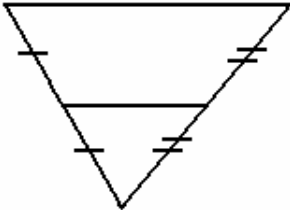
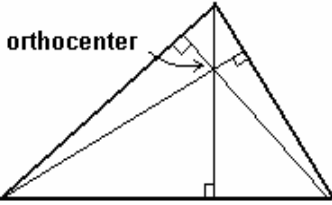
<i>Centroid</i>	<p>The centroid of the triangle is the point of congruency of the medians of the triangle.</p> 
<i>Chords</i>	A segment whose endpoints lie on the circle
<i>Circle</i>	The set of all points in a plane that are an equal distance (radius) from a given point (the center) which is also in the plane
<i>Circumcenter</i>	<p>A circumcenter is the point of concurrency of the perpendicular bisectors of a triangle.</p> 
<i>Circumference</i>	The distance around a circle
<i>Circumscribed</i>	<p>A circle is circumscribed about a polygon when each vertex of the polygon lies on the circle. (The polygon is inscribed in the circle.)</p> 
<i>Collinear points</i>	Points in the same plane that lie on the same line
<i>Complementary angles</i>	Two angles whose measures add up to 90 degrees

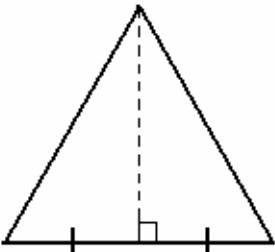
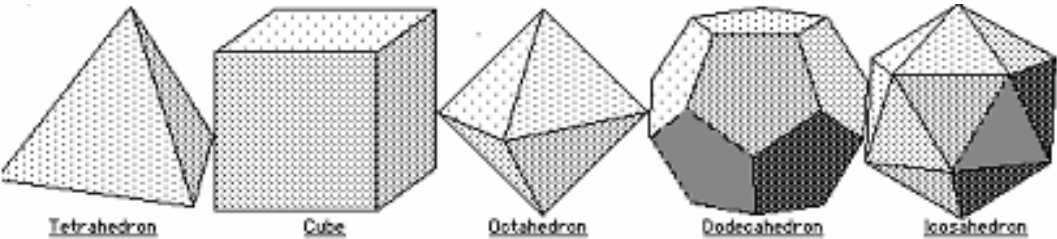
<i>Concentric circles</i>	Concentric circles lie in the same plane and have the same center
<i>Conditional statements</i>	A statement that can be written in the form "if p, then q" (Statement p is the hypothesis and statement q is the conclusion.)
<i>Cone</i>	A three dimensional figure with one circle base and a vertex <div style="text-align: center;">  </div>
<i>Congruent</i>	Having the same measure
<i>Conjecture</i>	Something believed to be true but not yet proven (an educated guess)
<i>Consecutive angles</i>	In a polygon, two angles that share a side <div style="text-align: center;">  </div>
<i>Consecutive sides</i>	In a polygon, two sides that share a vertex
<i>Contrapositive</i>	The contrapositive of a conditional statement ("if p, then q" is the statement "if not q, then not p")
<i>Converse</i>	The converse of the conditional statement interchanges the hypothesis and conclusion (“if p, then q, becomes “if q, then p”)

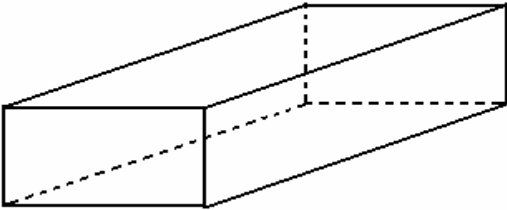
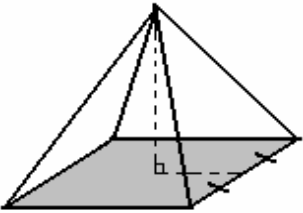
<i>Convex polygon</i>	A polygon in which no segment that connects two vertices can be drawn outside the polygon
<i>Coordinate geometry</i>	Geometry based on the coordinate system
<i>Coordinate plane</i>	A grid formed by two axes that intersect at the origin (The axes divided the plane into 4 equal quadrants.)
<i>Coplanar points</i>	Points that lie in the same plane
<i>Corollary</i>	A corollary of a theorem is a statement that can easily be proven by using the theorem.
<i>Corresponding parts</i>	A side (or angle) of a polygon that is matched up with a side (or angle) of a congruent or similar polygon 
<i>Cosine</i>	In a right triangle, the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse
<i>Cross-section</i>	A cross-section is the intersection of a solid and a plane.
<i>Cylinder</i>	A space figure whose bases are circles of the same size 
<i>Deductive reasoning</i>	Using facts, definitions, and accepted properties in a logical order to reach a conclusion or to show that a conjecture is always true
<i>Dilations</i>	Transformations producing similar but not necessarily congruent figures

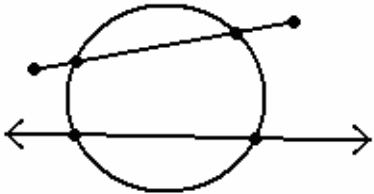
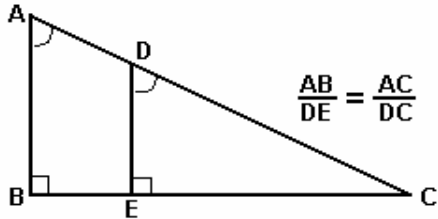
<i>Exterior angle of a polygon</i>	<p>An angle formed when one side of the polygon is extended (The angle is adjacent to an interior angle of the polygon.)</p>  <p>The diagram shows a pentagon with one side extended downwards. An arrow points to the angle formed between the extension and the adjacent side, labeled "Exterior Angle".</p>
<i>Geometric mean</i>	<p>If a, b, and x are positive numbers, and $a/x = x/b$, then x is the geometric mean of a and b.</p>
<i>Incenter</i>	<p>The incenter of a triangle is the point of congruency of the angle bisectors of the triangle.</p>  <p>The diagram shows a triangle with three dashed lines representing angle bisectors from each vertex. These lines intersect at a central point labeled "Incenter".</p>
<i>Inductive reasoning</i>	<p>A type of reasoning in which a prediction or conclusion is based on an observed pattern</p>
<i>Inscribed angle</i>	<p>An angle whose vertex is on a circle and whose sides are chords of the circle</p>  <p>The diagram shows a circle with a central dot representing the center. Two chords are drawn from a single vertex on the circumference to two other points on the circumference, forming an inscribed angle.</p>

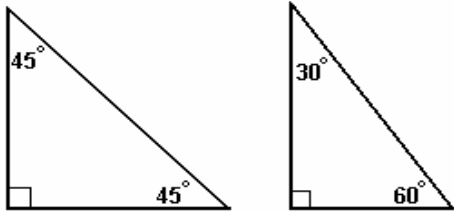

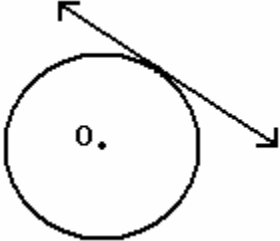
<i>Inscribed circle</i>	A circle is inscribed in a polygon if the sides of the polygon are tangent to the circle. 
<i>Inscribed polygon</i>	A polygon is inscribed in a circle if the vertices of the polygon are on the circle. 
<i>Interior angles of a polygon</i>	The inside angle of a polygon formed by two adjacent sides
<i>Inverse statement</i>	The inverse of the conditional statement ("if p, then q" is the statement "if not p, then not q")
<i>Irregular polygon</i>	A polygon where all sides and angles are not congruent
<i>Isometric drawings</i>	Drawings on isometric dot paper used to show 3-dimensional objects
<i>Isosceles triangle</i>	A triangle with at least two sides congruent
<i>Line of symmetry</i>	The line over which a figure is reflected resulting in a figure that coincides exactly with the original figure
<i>Linear pair of angles</i>	Two adjacent angles form a linear pair if their non-shared rays form a straight angle. 
<i>Matrix logic</i>	Using a matrix to solve logic problems


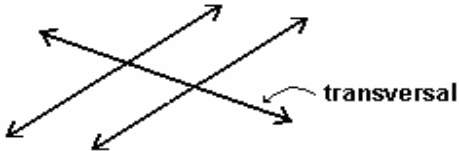
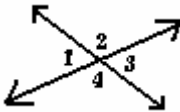
<i>Median of a triangle</i>	<p>A segment that has as its endpoints a vertex of the triangle and the midpoint of the opposite side</p> 
<i>Midpoint of a segment</i>	<p>The point that divides a segment into two congruent segments</p>
<i>Midsegment</i>	<p>A segment whose endpoints are the midpoints of two sides of a polygon</p> 
<i>Orthocenter</i>	<p>The orthocenter is the point of concurrency of the altitudes of a triangle.</p> 
<i>Orthographic drawings</i>	<p>An orthographic drawing is the top view, front view and right side view of a three-dimensional figure.</p>
<i>Parallel lines</i>	<p>Lines in a plane that never intersect</p>

<i>Parallelogram</i>	A quadrilateral with both pairs of opposite sides parallel
<i>Perimeter</i>	The distance around a polygon
<i>Perpendicular bisector</i>	The perpendicular bisector of a segment is a line, segment or ray that is perpendicular to the segment at its midpoint. 
<i>Perpendicular</i>	Two lines, segments, rays, or planes that intersect to form right angles
<i>Planes</i>	A flat surface having no boundaries
<i>Platonic solid</i>	A polyhedron all of whose faces are congruent regular polygons, and where the same number of faces meet at every vertex 
<i>Point</i>	A specific location in space
<i>Polygon</i>	A closed plane figure whose sides are segments that intersect only at their endpoints with each segment intersecting exactly two other segments
<i>Postulates</i>	A mathematical statement that is accepted without proof

<i>Prism</i>	<p>A three-dimensional figure--with two congruent faces called bases--that lies in parallel planes (The other faces called lateral faces are rectangles that connect corresponding vertices of the bases.)</p> 
<i>Pyramid</i>	<p>A three-dimensional figure with one base that is a polygon (The other faces, called lateral faces, are triangles that connect the base to the vertex.)</p> 
<i>Quadrilateral</i>	A four-sided polygon
<i>Radius</i>	A line segment having one endpoint at the center of the circle and the other endpoint on the circle
<i>Reflections</i>	Mirror images of a figure (Objects stay the same shape, but their positions change through a flip.)
<i>Regular octagon</i>	An octagon with all sides and angles congruent
<i>Regular polygon</i>	A polygon with all sides and angles congruent
<i>Rotations</i>	A transformation in which every point moves along a circular path around a fixed point called the center of rotation
<i>Scale drawings</i>	Pictures that show relative sizes of real objects

<p><i>Secants</i></p>	<p>A line, ray or segment that intersects a circle at two points</p> 
<p><i>Similarity</i></p>	<p>The property of being similar</p>
<p><i>Similar polygons</i></p>	<p>Two polygons are similar if corresponding angles are congruent and the lengths of corresponding sides are in proportion.</p> 
<p><i>Sine</i></p>	<p>In a right triangle, the ratio of the length of the leg opposite the angle to the length of the hypotenuse</p>
<p><i>Slope</i></p>	<p>The ratio of the vertical change to the horizontal change</p>
<p><i>Slope-intercept form</i></p>	<p>A linear equation in the form $y = mx + b$, where m is the slope of the graph of the equation and b is the y intercept</p>

<i>Special right triangles</i>	<p>A triangle whose angles are either 30-60-90 degrees or 45-45-90 degrees</p> 
<i>Spheres</i>	<p>The set of all points in space equal distance from a given point</p> 
<i>Standard form of an equation</i>	<p>The form of a linear equation $Ax + By = C$ where A, B, and C are real numbers and A and C are not both zero Ex. $6x + 2y = 10$</p>
<i>Supplementary angles</i>	<p>Two angles whose measures add up to 180 degrees</p>
<i>Surface area</i>	<p>The area of a net for a three-dimensional figure</p>
<i>Tangent</i>	<p>In a right triangle, the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle</p>
<i>Tangent to a circle</i>	<p>A line in the plane of the circle that intersects the circle in only one point</p> 

<i>Tessellate</i>	A pattern of polygons that covers a plane without gaps or overlaps 
<i>Theorems</i>	A conjecture that can be proven to be true
<i>Transformation</i>	A change made to the size or position of a figure
<i>Translation</i>	A transformation that slides each point of a figure the same distance in the same direction
<i>Transversal</i>	A line that intersects two or more other lines in the same plane at different points 
<i>Triangle Inequality Theorem</i>	The sum of the lengths of any two sides of a triangle is greater than the lengths of the third side.
<i>Trigonometric ratios</i>	The sine, cosine and tangent ratios
<i>Venn diagram</i>	A display that pictures unions and intersections of sets
<i>Vertical angles</i>	Non-adjacent, non-overlapping congruent angles formed by two intersecting lines (They share a common vertex.)  <p>$\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are vertical angles.</p>
<i>Volume</i>	The number of cubic units needed to fill a space