

# Topics in Discrete Mathematics

## Mathematics Curriculum Framework

Revised 2004  
Amended 2008

Course Title: Topics in Discrete Mathematics  
 Course/Unit Credit: 1  
 Course Number:  
 Teacher Licensure: Secondary Mathematics  
 Grades: 9-12

### Topics in Discrete Mathematics

Topics in Discrete Mathematics is designed for students who have successfully completed Geometry and Algebra II. This course will build upon concepts presented in Algebra II and Geometry to encourage higher order thinking with emphasis on applications. These students will represent, model, and analyze authentic mathematical situations. Matrices, *optimization*, measurement, exponential functions, and data analysis will be explored in Topics in Discrete Mathematics. Arkansas teachers are responsible for integrating appropriate technology into the coursework for Topics in Discrete Mathematics.

Strand	Content Standard
Matrices	1. Students will use matrices to analyze data to solve real-world problems.
<i>Optimization</i>	2. Students will use various techniques to solve <i>optimization</i> problems.
Measurement	3. Students will apply various measurement techniques to solve real-world problems.
Exponential Functions	4. Students will extend algebraic skills to solve real-world problems involving exponential/logarithmic functions.
Data Analysis	5. Students will analyze data using various statistical tools.

Strand: Matrices

Standard 1: Students will use matrices to analyze data to solve real-world problems.

MA.1.TDM.1	Collect and interpret data in a matrix and perform operations to solve real-world problems, with and without technology
MA.1.TDM.2	Solve real-world problems involving systems of linear equations using matrices (e.g., inverses, <i>augmented</i> , <i>Cramer's rule</i> )
MA.1.TDM.3	Find and use the inverse of a matrix to solve real-world problems (e.g., cryptology)
MA.1.TDM.4	Organize and use <i>transition matrices</i> to solve probability problems that link present events to future events, with or without technology (e.g., consumer trends, polling trends, board games, weather trends)

Strand: *Optimization*

Standard 2: Students will use various techniques to solve *optimization* problems.

OP.2.TDM.1	Graph systems of linear inequalities with multiple <i>constraints</i> and identify vertices of the <i>feasible region</i>
OP.2.TDM.2	Model and solve real-world problems using <i>linear programming</i> (e.g., maximum profit/minimal cost, investments, agriculture, manufacturing, banking)
OP.2.TDM.3	Interpret the meaning of the minimum or maximum value in terms of the <i>objective function</i>
OP.2.TDM.4	Model and solve real-world problems involving <i>optimization</i> of area and volume

Strand: Measurement

Standard 3: Students will apply various measurement techniques to solve real-world problems.

ME.3.TDM.1	Solve problems using <i>dimensional analysis (factor-label method)</i> (e.g., construction, medical, metric, standard to metric, rate conversions)
ME.3.TDM.2	Use sine, cosine, and tangent ratios to determine lengths of sides and angle measures of right triangles for real-world problems (e.g., angles of elevation and depression and various distances)
ME.3.TDM.3	Use laws of sine and cosine to determine lengths of sides, measures of angles, and area of triangles for real-world problems (e.g., <i>Heron's formula</i> )
ME.3.TDM.4	Calculate the area of two-dimensional <i>composite figures</i>
ME.3.TDM.5	Calculate the surface area and volume of three-dimensional <i>composite figures</i>

Strand: Exponential Functions

Standard 4: Students will extend algebraic skills to solve real-world problems involving exponential/logarithmic functions.

EF.4.TDM.1	Draw and recognize the graphs of logarithmic and exponential functions, with and without appropriate technology
EF.4.TDM.2	Apply properties of logarithms to convert and solve logarithmic (common and natural) and exponential equations
EF.4.TDM.3	Use the <i>change of base formula</i> to simplify and evaluate logarithmic expressions, using technology
EF.4.TDM.4	Recognize and apply properties of exponential functions to solve real-world problems (e.g., compound interest, amortization, annuities, appreciation, depreciation)
EF.4.TDM.5	Recognize and apply properties of logarithmic functions to solve real-world problems (e.g., Richter scale pH, decibel scale, bacterial growth, radioactive decay, Newton's Law of Cooling)

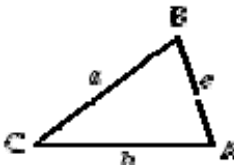
Strand: Data Analysis

Standard 5: Students will analyze data using various statistical tools.

DA.5.TDM.1	Read, interpret, and analyze graphical representations of data used in various contexts (e.g., science reasoning, newspaper graphs)
DA.5.TDM.2	Identify <i>biases</i> that affect the <i>validity</i> of a data set
DA.5.TDM.3	Collect, analyze, and compare data sets using <i>five-number summary</i>
DA.5.TDM.4	Investigate and analyze the characteristics of <i>normal</i> and <i>skewed distributions</i>
DA.5.TDM.5	Determine and interpret the measures of spread of a data set (e.g., <i>standard deviation, range, percentiles, variance</i> )

Glossary for Topics in Discrete Mathematics

<p>Augmented matrix</p>	<p>A matrix form of a linear system of equations obtained from the coefficient matrix as shown below (It is created by adding an additional column for the constants on the right of the equal signs. The new column is set apart by a vertical line.)</p> $\text{System: } \begin{array}{l} 2x - 3y = 8 \\ 4x + 5y = -1 \end{array} \quad \text{Augmented Matrix: } \left[ \begin{array}{cc c} 2 & -3 & 8 \\ 4 & 5 & -1 \end{array} \right]$
<p>Bias</p>	<p>A systematic error introduced into sampling or testing by selecting or encouraging one outcome or answer over others</p>
<p>Change of base formula</p>	<p>A formula that allows you to rewrite a logarithm in terms of logs written with another base (This is especially helpful when using a calculator to evaluate a log to any base other than 10 or <math>e</math>.)</p> <p>Assume that <math>x</math>, <math>a</math>, and <math>b</math> are all positive. Also assume that <math>a \neq 1</math>, <math>b \neq 1</math>.</p> $\log_a x = \frac{\log_b x}{\log_b a}$ <p>Change of base formula:</p>
<p>Composite figure</p>	<p>A geometric figure that is made up of two or more shapes</p>
<p>Constraint</p>	<p>A statement or inequality that defines the <i>feasibility region</i></p>

Cramer's rule	<p>A method for solving a linear system of equations using determinants (<i>Cramer's rule</i> may only be used when the system is square and the determinant of the coefficient matrix is not 0.)</p> <p style="text-align: center;"> <math>x = 7, y = 8, z = 10</math>  <b>System:</b> <math>x + 2y + 3z = 11</math>  <math>4x + 5y + 6z = 12</math> </p> <p style="text-align: center;"><b>Solution by Cramer's rule:</b></p> $x = \frac{\begin{vmatrix} 11 & -1 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}} = \frac{-99}{-12} = \frac{99}{12} = \frac{33}{4}$ $y = \frac{\begin{vmatrix} 1 & 11 & 1 \\ 4 & 12 & 6 \\ 1 & 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}} = \frac{-99}{-12} = \frac{99}{12} = \frac{33}{4}$ $z = \frac{\begin{vmatrix} 1 & 2 & 11 \\ 4 & 5 & 12 \\ 1 & 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}} = \frac{-99}{-12} = \frac{99}{12} = \frac{33}{4}$
Dimensional analysis	See <i>factor-label method</i> .
Factor-label method	The sequential application of conversion factors expressed as fractions and arranged so that any dimensional unit appearing in both the numerator and denominator of any of the fractions can be cancelled out until only the desired set of dimensional units is obtained; <i>dimensional analysis</i>
Feasible region	A set of points which satisfies every <i>constraint</i>
Five-number summary	A data set that consists of (1) the minimum (smallest observation), (2) the lower quartile or first quartile (which cuts off the lowest 25% of the data), (3) the median (middle value), (4) the upper quartile or third quartile (which cuts off the highest 25% of the data), and (5) the maximum (largest observation)
Heron's formula (or Hero's formula)	<p>A formula for the area of a triangle used when the lengths of all three sides are known</p> <p style="text-align: center;"><b>Heron's Formula</b></p> <p>If <math>s = \frac{a+b+c}{2}</math>, then the area = <math>\sqrt{s(s-a)(s-b)(s-c)}</math>.</p> <p>Hence <math>s = \frac{a+b+c}{2}</math> is called the <i>semiperimeter</i>.</p> 

Linear programming	A method for finding the maximum or minimum value of a function in two variables subject to given <i>constraints</i> on the variables
Normal distribution	A pattern for the distribution of a set of data which follows a bell shaped curve; sometimes called the Gaussian distribution
Objective function (or objective quantity)	An algebraic expression in two or more variables describing a quantity that must be maximized or minimized
Optimization	The process by which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within a domain
Percentile	A value on a scale of one hundred that indicates the percent of a distribution that is equal to or below it
Range	The difference between the maximum and minimum in a set of data
Skewed distribution	A non-symmetric distribution that may be skewed right or skewed left
Standard deviation	Measures how widely spread the values in a data set are (If many data points are close to the mean, then the <i>standard deviation</i> is small; If many data points are far from the mean, then the <i>standard deviation</i> is large.)
Transition matrix	A square matrix describing the probabilities of moving from one state to another in a dynamic system; matrix that represents the probabilities from one situation to another situation in one step
Validity	The degree to which an observed result can be relied upon and not attributed to random error in sampling
Variance	The square of the <i>standard deviation</i>